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OPTICAL FRINGE ANALYSIS TECHNIQUES

Dr. T. L. Williamson C. R. Lane



Electro-Optics Technology Branch Electronic Technology Division

June 1979

TECHNICAL REPORT AFAL-TR-79-1115, Vol I

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ABSTRACT (Continue on reverse side it necessary and identity by block number)  The use of specklegrams for performing certain types of NDI and NDT has been known for some time yet a method for extracting the data quickly and chear has eluded specklegram users. This report discusses a new technique for performing the data reduction of the large amounts of data obtained through speckle interferograms. This new technique correlates an optical mask on a fringe pattern to generate a single line scale proportional to the frequency of the cosine fringe pattern when integrated orthogonally to the fringe line.	17. DISTRIBUTION STATEMENT (of the abetract entered in Block 20, if different from the state of	pes of NDI and NDT has bee the data quickly and cheap ses a new technique for perf data obtained through lates an optical mask on a oportional to the frequence

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# SECTION I

Considerable interest has been kindled over the past several years in applying holographic interferometry and specklegram interferometry to the areas of NDT (non-destructive testing) and NDI (non-destructive inspection). The results of this application are usually less successful than desired. The primary reason for this is that the interferometry is so sensitive that the analyst is overwhelmed with data and must spend considerable time going over this data to deduce what it all means. It is ,thus, essential that techniques be developed to reduce the fringe patterns to a lesser quantity of more meaningful data.

The potential for automated reduction of the speckle-gram fringe data has been demonstrated and is presently being pursued under a joint AFFDL/AFAL program. This program utilizes conventional digital processing techniques to perform the data reduction. The required system for this approach is very complex and expensive. A technique is needed that is less complex and hence requires less expensive equipment for implementation. The following discussion is the result of a first attempt to take a more general look at the problem with the goal of a more specific

and simple solution. This first look has resulted in some promising approaches to the solution of the specklegram fringe analysis using optical techniques to do the majority of the work. This results in lessening the requirements for data storage and hence less expensive computer equipment.

## SECTION II SPECKLEGRAM

The specklegram under consideration is a photographic recording of a double exposure of an object. The speckle is the result of using laser illumination on an object with a diffuse surface. If the object has undergone a surface displacement between the exposures either due to external forces or gross displacement, this information will be recorded in the specklegram. The ensuing discussion involves the recording and subsequent readout of this displacement information.

The speckle is in effect a random sampling function whose sample size is small compared to the areas under investigation. The size of the speckle can be controlled by the recording optics. The speckled image of an object can be represented by

$$\sum_{x_{i},y_{j}}I(x_{i},y_{j}) \delta(x-x_{i},y-y_{j})$$
 (1)

where I(x,y) represents the unspeckled image of the object and the delta function represents the individual speckle. The delta function is used since the speckle is small compared to the minimum resolution of the image that is being investigated.

The second exposure on the film can be represented by the similar expression

$$\sum_{x_{k},y_{1}} I(x_{k},y_{1}) \delta(x-x_{k},y-y_{1})$$
 (2)

where the summation is now taken over the second set of speckles. For small surface displacements, the positions of the speckles on the second image are related to those of the first by

$$x_k = x_i + \Delta x$$
;  $y_1 = y_i + \Delta y$  (3)

where  $\Delta x$  and  $\Delta y$  represent the displacement in the two coordinate directions. The field for the second exposure can then be written as

$$\sum_{x_{i},y_{j}} I(x_{i} + \Delta x, y_{j} + \Delta y) S(x - x_{i} - \Delta x, y - y_{j} - \Delta y)$$
 (4)

The photographic recording process is a square law process resulting in a total exposure for a unit time given by

$$E(x,y) = \sum_{x_{i},y_{j}} \left[ [I(x_{i},y_{j}) \delta(x - x_{i}, y - y_{j})]^{2} + \right]$$

$$\left[I\left(x_{i}^{+}\Delta x,y_{j}^{+}\Delta y\right)\delta\left(x-x_{i}^{-}\Delta x,y-y_{j}^{-}\Delta y\right)\right]^{2}$$

The summation has been taken outside of the brackets since the cross product of delta function is zero unless the arguments are equal.

Since the specklegram is being used only for a point to point comparison of the surface displacement, the area of interest at any one time is small compared to the detail of the image. For this reason, the image function can be considered a constant over each area of interrogation. Thus, over a small area the exposure function can be represented by

$$E(x,y) = K \sum_{x_{i},y_{j}} (\delta(x-x_{i},y-y_{j}))^{2} + (\delta(x-x_{i}-\Delta x,y-y_{j}-\Delta y))^{2}$$
(6)

where K is the constant value for the image function over the area. The exposed film consists of a collection of delta function type sampling functions whose positions are determined by the relationship of equation 6. The granular nature of the photographic emulsion dictates that either the silver particle exists at a given location or it doesn't, therefore, the squaring of the delta functions has no significance in the resultant image that is recorded. The resultant recorded image in the small area can be represented by

$$E(x,y) = K \sum_{x_1,y_j} (\delta(x-x_1,y-y_j) + \delta(x-x_1-\Delta x,y-y_j-\Delta y)) (7)$$

The resultant specklegram will then consist of an array of speckle sampling functions (represented by the delta functions) whose positions reflect the displacement that occured between exposures. Each of these speckles will be recorded in the form of an absorptive silver particle or collection of silver particles. It will be assumed for simplicity that the film will be completely absorptive at a speckle position. This assumption does not affect the form of the result, but rather only the multiplicative constant. The transmission of the film can then be written in the form

$$T(x,y) = 1 - \sum_{x_{i},y_{j}} (\delta(x-x_{i},y-y_{j}) + \delta(x-x_{i}-\Delta x,y-y_{j}-\Delta y))$$

$$= 1 - (\delta(x,y) + \delta(x-\Delta x,y-\Delta y)) + \sum_{x_{i},y_{j}} \delta(x-x_{i},y-y_{j})$$
(8)

where \* represents the convolution operation.

The specklegram is analyzed by illuminating each individual area by a small collimated laser beam and observing the resultant far field pattern. The amplitude of the field existing in the far field region of the area being analyzed will be the Fourier transform of the field existing at the exit aperture of the film. If the field used for the illumination is of unit amplitude then the

exit field will be represented by the expression of equation 8. The field in the far field region will then be obtained by taking the Fourier transform of this expression.

$$E_{ff}(x_1,y_1) = \mathcal{F}[T(x,y)]$$
 (9)

On performing the Fourier transform term by term the field that exists in the far field can be represented by the expression

$$E_{ff}(x_{1},y_{1}) = \delta(x_{1},y_{1}) + 2e^{-i(\Delta x x_{1} + \Delta y y_{1})}$$

$$Cos[(\Delta x x_{1} + \Delta y y_{1})/2] \sum_{x_{1}'y_{1}'} \delta(x_{1} - x_{1}', y_{1} - y_{1}') (10)$$

The first term represents the "D-C" or transmitted beam and the second term is a set of speckled cosine fringes. The period of the cosine fringe pattern represents the displacement that occured between exposures and is the desired parameter to be measured. The series of delta functions is the transform of the series that existed in the exit aperture and is merely the speckle that appears in the observed pattern. The phase term will not be of importance for the cases when the intensity of the far field is observed rather than it's amplitude. A typical far field intensity pattern is shown in Figure 1.

Several observations can be made at this point which could prove to be of great benefit in establishing any generalized techniques of specklegram analysis. The speckle will be present at all stages of the analysis unless averaging techniques are applied to eliminate it. It's size can be controlled through proper choice of both the construction and analysis optics so that it's effect may be minimized in the analysis. The far field pattern is not general, but rather a series of cosine fringes. For this reason, a generalized analysis of the pattern is not required. The desired information can be obtained from only the fringe spacings and their orientation.

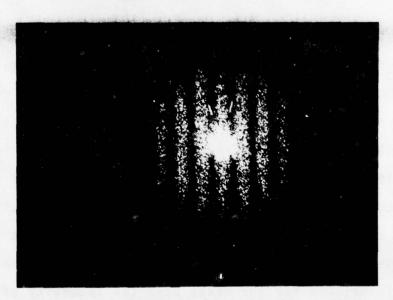


FIGURE 1: Typical far field intensity pattern.

## SECTION III SPECKLEGRAM FRINGE ANALYSIS

In this section a technique of fringe analysis will be discussed that relies on the knowledge of the form of the far field pattern that was obtained in the preceeding section.

Since it was determined in the preceeding section that the fringe system resulting from a specklegram is, or at least can be made to be, a series of speckled cosine fringes, the following analysis applies. the far field pattern is an even function, the Fouriercosine transform can be used to determine the relationship between fringe pattern and the speckled double image. In the same manner the Fourier cosine transform can be used to analyze the fringe pattern itself. The fringe pattern is of the form A Coskox , where A is the arbitrary amplitude and ko is the radian frequency of the pattern. In fact if the expression for the total fringe distribution is expanded in a cosine series only one term would have a non-zero coefficient, that corresponding to the proper spatial frequency. The general form for the coefficients of the cosine series expansion would be

$$a_{x} = \int_{-\infty}^{\infty} ACos(k_{o}x)Cos(k_{x}x) dx$$
 (11)

This operation can easily be implemented, since it consists merely of multiplying the fringe pattern by a cosine function and integrating over the variable x. From this it is evident that if a mask were constructed with transmission given by  $Cos(k_1(y+b))$  then the product of the incident fringe pattern (when properly aligned) and this transmission function would be the integrand of equation 11. The integral of the expression over the x variation would be a function of y and would represent the coefficient of the corresponding cosine term in the expansion. The expression would be

$$a_{\chi}(y) = \int_{-\infty}^{\infty} ACos(k_0 x)Cos(k_1 (y+b)) dx$$
 (12)

This expression would have a non-zero value only when

$$k_0 = k_1(y+b) \tag{13}$$

Thus, the y value for which a non-zero value was obtained would be a measure of the fringe spacing.

In reality the aperture over which the fringe pattern exists is not infinite, resulting in the expression

$$a_{x}(y) = \int_{-\infty}^{D} ACos(k_{0}x)Cos(k_{1}(y+b)) dx$$

$$= \int_{-\infty}^{\infty} P_{D}(x)ACos(k_{0}x)Cos(k_{1}(y+b))dx$$
(14)

where 2D is the aperture diameter in the x direction and  $P_D(x)$  takes on the value of 1 for -D < x < D and is zero elsewhere. The effect of this non-infinite aperture is that instead of a delta function occurring at the value of y for which equation 13 holds the function  $a_X(y)$  will be the Fourier transform of  $P_D(x)$  and will be centered at the previous value of y. This extends in the y direction since the y variation is a measure of the x spatial frequency.

$$a_{\chi}(y) = \frac{2 \sin(k_1 D(y+b))}{k_1 (y+b)}$$
 (15)

This function will determine the ultimate resolution of the technique as a function of the aperture size or the number of fringes analyzed.

In reality the transmission of the mask does not have to be a pure cosine function. It is only required that it have a fundamental cosine frequency sufficiently strong that the location of it's maximum can easily be determined on the y-axis. Therefore, even a square wave variation would be adequate. The spatial variation of such a transmission mask would appear as shown in Figure 2.

The product of the mask function of figure 2 with fringe system of equally spaced lines is shown in Figure 3. The resultant Moire' fringe systems gives a visual

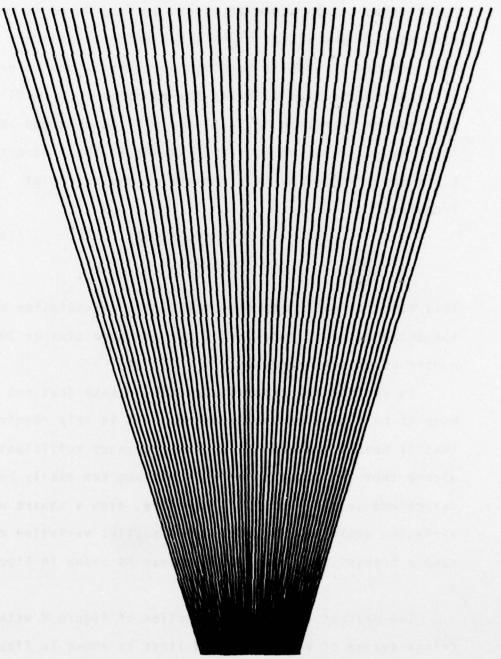


FIGURE 2 : MASK FRINGE VARIATION

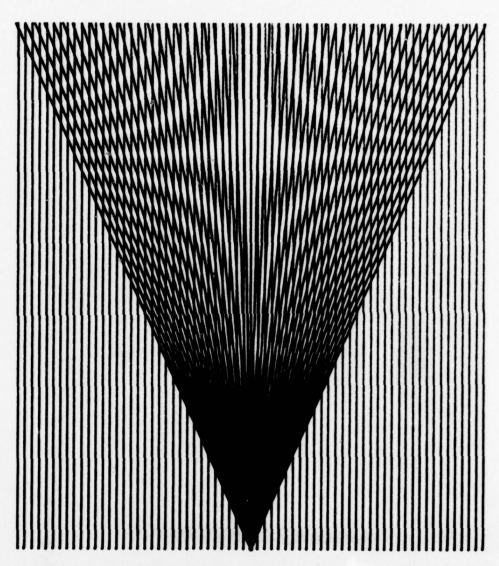


FIGURE 3: Product of Mask Transmission and Constant Fringe Pattern.

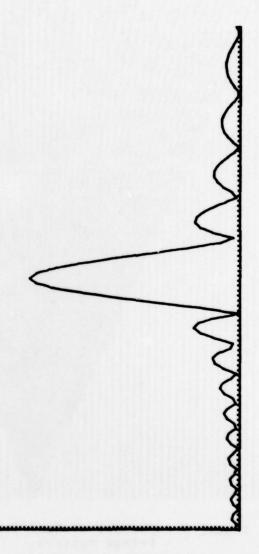


FIGURE 4 : Correlation of Pattern of Figure 3.

measure of the correlation that is taking place. The pattern in this form would not be easy to measure with an E-O device. Figure 4 shows the function that is obtained when the function in figure 3 is integrated over the x coordinate (horizontal). This function not only readily shows the location of the y value for maximum correlation but also shows the shape of the aperture induced spread function. The fringe patterns as well as the integration in these figures were simulated on the Hewlett-Packard 9820 calculator.

The preceeding discussion has outlined a rather simple approach to determining the spatial frequency of the cosine fringe pattern. The procedure used does not require the implementation of a general Fourier transform operation. As will be shown in the following section this technique is easily implemented optically.

## SECTION IV

### EXPERIMENTAL VERIFICATION

In order to demonstrate the concept of fringe analysis discussed in the previous section optically, a reference specklegram was reconstructed giving the speckled cosine fringe pattern shown in Figure 5. This fringe system was imaged onto a standard mask (Edmund Scientific Moire' Pattern #4) having a square wave transmission characteristic whose spacing varied proportionally to the radius. The resultant product pattern is shown in Figure 6. Note that the Moire' fringes can be seen in the same form as those of Section III.

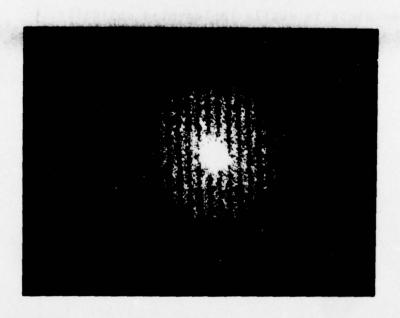


FIGURE 5 : Cosine Fringe System

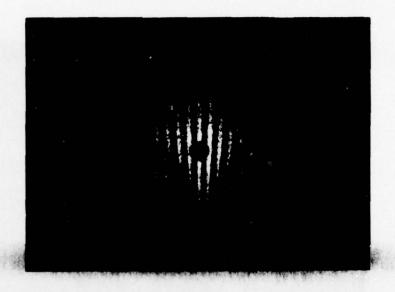


FIGURE 6 : Product of Cosine Fringe System with

Transmission Mask.

The resultant field is then integrated along the direction of the fringe propogation by means of a cylindrical lens. The intensity pattern that is obtained will then give a measure of the position in the other coordinate direction where the correlation between the mask and the fringe pattern is a maximum. This position can then be related to the spatial frequency of the mask at that position and the resultant fringe spacing obtained. Figure 7 shows the result of this integration on the nattern of Figure 6.

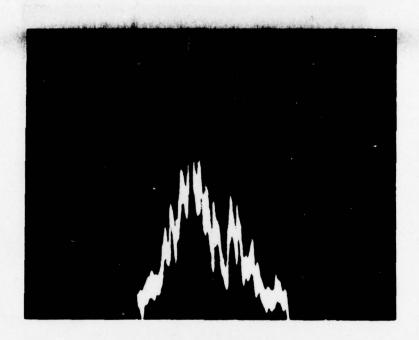


FIGURE 7: Resultant Correlation Function Relating the Spatial Frequency of the Fringes to Those of the Mask.

The correlation of Figure 7 was obtained by taking the intensity profile of the image on an image analyzer.

This experiment has demonstrated that the fringe analysis can be implemented simply through the use of a transmission mask and a cylindrical lens.

#### SECTION V

### CONCLUSIONS

The preceeding discussion has demonstrated that there are techniques of analyzing the fringes obtained from specklegrams that are easily implemented optically. It is not claimed that the technique discussed in the preceeding sections is the only or the best solution to the problem, but rather that it is one solution and it does work.

The procedure that has been outlined takes advantage of the known characteristics of the fringe pattern that is being investigated. The important features are that the fringe pattern is a cosine function and that it is always centered on the D-C spot, or at least can be made to be centered. These features permit the analysis to be made through a rather simple correlation technique with one transmission mask. The resultant correlation function could then be measured by a detector array to give a completely automated readout. The fringe pattern from the specklegram could be rotated by an optical system using a dove prism before correlation to obtain the angular information.

Before the techniques discussed here could be implemented, it will be necessary to do a more thorough analysis of the procedures to optimize the techniques. Some of the areas that need further consideration are;

- Perform an error analysis of the procedure to determine the accuracy as a function of the number of fringes available in the aperture and the fringe contrast.
- 2. Optimize the specklegram recording procedure.
- 3. Develop techniques of measuring the fringe spacing for maximum correlation and it's associated angle.
- 4. Determine the potential of combining the correlation and integration steps into a single holographic element.

